Journal of Engineering Physics and Thermophysics, Vol. 76, No. 3, 2003

MATHEMATICAL MODELING OF THE PROCESSES OF DEFORMATION OF SOILS WITH TIME

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UDC 539.376:631.43:001.573

It has been suggested that the rheological properties of soils be modeled by the integral Volterra equation of the second kind of the nonlinear heredity theory and by the differential equation which, under certain conditions, approximately replaces the adopted integral equation. Parameters of these governing equations have been found from experimental data for a number of soils. The processes of creep of soils have been studied.

At present, the problem of conservation and improvement of soil fertility is one of the most pressing environmental problems; large numbers of scientists are engaged in solving it. However, a number of negative anthropogenic effects impede the conservation of soil fertility.

There are a great number of works devoted to elucidating relations between loads and deformations in soils and to studying changes in the properties of soils under the effect of outer loads [1–3], although this complex problem has not yet been properly solved. Computing methods based on the results of theoretical and experimental studies of the processes occurring in deformation of soils must play an important role in the development of a complex of measures aimed at conserving soil fertility.

The accuracy of the computing methods of determination of the compaction factors of soils under the effect of outer loads depends first of all on the choice of mathematical models of deformation of soils.

The governing equations of the mechanics of grounds and, in particular, of soils, fall into two types: (1) equations of the relation between stresses and deformations (not involving time) and (2) equations allowing for changes in the stresses and deformations with time. Equations of the first type describe the curves obtained by stepwise static loading of soils in compression, shear, and punch tests where only the deformation loads which are stabilized (conventionally) in each step are recorded. Equations of the second type characterize the curves which can be obtained in various modes and under various conditions of deformation with stresses and deformations being recorded at different fixed instants of time.

Use of equations of the first type has a number of substantial drawbacks. The dependences of the form $\sigma = \sigma(h)$ or $\sigma = \sigma(\varepsilon)$ model and allow one to take into account only residual deformations, whereas in water-unsaturated soils reversible deformations also take place. The values of both reversible and residual deformations of various soils depend on the rate of application of the load and the time of its action. However, relations of the first type do not reflect this fact and do not make it possible to take into account in calculations the effect of loads which change with time according to different laws.

The above drawbacks can justifiably be overcome by using governing relations of the second kind, i.e., rheological equations or equations of viscoelasticity theory.

To each physical state of soil, depending on its humidity, there correspond its own laws governing deformation under the effect of compression load. At moisture contents w, which are lower than the total moisture capacity w_{tot}, deformations arising in the soil due to its loading consist of irreversible (residual) and reversible parts. Under the effect of loading, the soil packs and strengthens. As a result of the outer load transferred to the soil by the pressure of the die an increase of setting caused by the increased load gradually damps and deformations become stabilized. Residual structural deformations of soils can conventionally be referred to viscous deformations, since they arise under the effect of any small forces and the rates of these deformations increase with increase in the acting forces.

After the removal of the outer load, reversible deformations of the soil appear; these deformations consist of elastic and viscous deformations. Reversible deformations, which occur during a certain period of time, i.e., are char-

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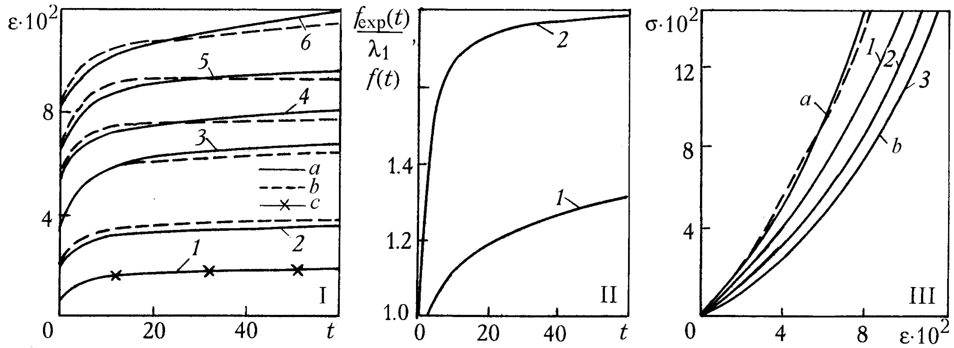

Fig. 1. Creep in compression of the silt-loam soil (density ρ = 1.34 g/cm^3, humidity w = 17%): I, experimental (a) and calculated (b) curves of soil creep; 1) σ = 0.0125; 2) 0.025; 3) 0.05; 4) 0.075; 5) 0.1, 6) 0.15 MPa; coincidence of experimental and calculated curves (c); II, experimental (1) and theoretical (2) curves of soil compliance [1), $f_{\exp}(t)/\lambda_1$; 2) (5)]; III, experimental isochrones [1) t = 3, 2) 10; 3) 60]; a and b, calculated curves $\sigma = \varphi_0(\varepsilon)$ (dashed curve — approximation) and $\sigma = \varphi_\infty(\varepsilon)$, respectively; curves 3 and b coincide. t, sec.

acterized by the aftereffect (viscous deformations), have relatively higher values than elastic deformations [3]. The present work is devoted to the study and mathematical modeling, with account for the time factor, of laws governing the deformation of soils which are in the state described above (characterized by the values $w < w_{\text{tot}}$).

The equations of the Maxwell and Kelvin models refer to the most simple governing rheological equations [4]. These equations describe the properties of ideal viscoelastic media. These and some other equations of similar structure have been used in a number of works [2] as governing relations for soils. However, the appropriateness of the Kelvin and Maxwell models to the properties of real soils of certain granulometric composition and physical state was not confirmed experimentally.

A more general theory which allows modeling of laws governing the deformation of media with viscoelastic properties is the Boltzmann–Volterra heredity theory of viscoelasticity [5]. We studied the rheological properties of a number of loamy, sandy-loam, and sandy soils at different values of their density and humidity in order to find the corresponding governing equations based on this theory. To do this we processed a number of experimental sets of creep curves and curves of stress relaxation in soils obtained in compression and shear. We used the experimental data from [6–8] and others. As an example, Fig. 1, I gives the sets of creep curves $\varepsilon_i(t)$ (i = 1, 2, ..., r) obtained on compression of silt-loam soil in the compression device [6].

The results of the processing of experimental data showed that the laws governing compression and shear deformations of a number of soils with time are to a high degree of accuracy modeled by the linear integral Volterra equation of the second kind with a nonlinear free term

$$\varphi_0(\varepsilon) = \sigma(t) + \int_0^t K(t-\tau)\,\sigma(\tau)\,d\tau \tag{1}$$

and the Koltunov kernel

$$K(t) = \frac{\exp(-\beta t)}{t} \sum_{n=1}^{\infty} \frac{[A\Gamma(\alpha)]^n\, t^{\alpha n}}{\Gamma(\alpha n)}, \quad 0 < \alpha < 1\,. \tag{2}$$

The resolvent of Eq. (1) with kernel (2) is the function

$$T(t) = A \exp(-\beta t)\, t^{\alpha - 1} \tag{3}$$

which is the Rzhanitsyn kernel. The nonlinear function $\varphi_0(\varepsilon)$ describes the curve of instantaneous deformation (at t = 0).

Assuming in (1) $\sigma(t) = \sigma_i$ = const, we obtain the equations of creep curves

$$\varphi_0(\varepsilon) = \sigma_i f(t), \quad i = 1, 2, ..., r, \tag{4}$$

where

$$f(t) = 1 + \int_0^t K(t - \tau)\, d\tau \tag{5}$$

is the compliance function.

Relation (4) reflects the similarity of isochrones, i.e., the curves $\sigma = \varphi_j(\varepsilon)$, which correspond to various fixed instants of time $t = t_j$ (j = 0, 1, 2, ...). A combination of the similarity factors λ_j shifting the isochrones labeled $t = t_j$ to the curve of instantaneous deformation $\sigma = \varphi_0(\varepsilon)$ represents experimental values $f_{\exp}(t)$ of the function $f(t)$.

The graphs of the functions $f_{\exp}(t)/\lambda_1$ and $f(t)$ corresponding to the creep curves (Fig. 1, I) are shown in Fig. 1, II, while the curve $\sigma = \varphi_0(\varepsilon)$ found from these data is shown in Fig. 1, III.

For a number of soils we determined the numerical values of the parameters of kernel (2) of the integral rheological equation (1) using the technique suggested by Koltunov [5] and also found the numerical values of the functions $\sigma = \varphi_0(\varepsilon)$. The quantities obtained were used to construct computation creep curves of soils (Fig. 1, I). Certain experimental and calculated curves coincide in the scale of the figure. For the creep curves the mean value of relative deviations δ of the calculated data from the experimental ones, which was computed by processing 62 results of measurement, is δ_{mean} = 2.4%, and the root-mean-square (standard) deviation for δ is 3.38%. The results obtained indicate the adequacy of modeling the experimentally found rheological properties by Eq. (1) with kernel (2), with (1) describing the laws governing the deformations of soils with time very accurately.

At stresses $\sigma < \sigma_{st}$ we can find the curve of the limiting state $\sigma = \varphi_\infty(\varepsilon)$ — the isochrone labeled $t = \infty$. The curves $\varphi_0(\varepsilon)$ and $\varphi_\infty(\varepsilon)$ bound the entire possible region of soil deformation from above and below, respectively (Fig. 1, III).

Use of (1) allows one to complement and generalize the results of earlier studies of the laws governing the deformations of soils [3]. Indeed, on the basis of (4) any governing equation of the first type can be considered as the equation of the isochrone labeled $t = t_{stab}$, where t_{stab} is the time of conventional stabilization of deformation in each step of loading in static tests without regard for the time factor.

To solve a number of practical problems it is of importance to find a simpler governing equation, as compared to (1), which approximately replaces it at small t.

We approximate, at $t \in [0; t_1]$, expression (5) by the linear function $f(t) = 1 + pt$, where t_1 is a certain rather small value of time. In this approximation, $K(t) \approx p$ = const. It is known from experiments that at rather small values of σ deformations and stresses are almost proportional, and the curve $\sigma = \varphi_0(\varepsilon)$ has a form close to a straight line. Let $\varepsilon \in [0; \varepsilon_1]$, where ε_1 is a certain fixed small value of deformation. With the indicated approximations of the functions $K(t)$ and $\sigma = \varphi_0(\varepsilon)$ the integral equation (1) at comparatively small t and ε can approximately be replaced by the differential equation

$$\frac{d\sigma}{dt} + p\sigma = q\,\frac{d\varepsilon}{dt}. \tag{6}$$

The limits of applicability of Eq. (6) to specific soils must be found from experimental data.

Thus, we showed the interrelation of two types of rheological models of deformation of soils, viz., integral and differential equations.

Equation (6) was suggested for modeling the laws governing the deformation of compacting soils also on the basis of a theoretical analysis of the experimentally found deformation properties of soils studied in a number of works [3].

The equations which describe the laws governing the deformation of an ideal viscoelastic Maxwell model and Eq. (6) are similar in structure but differ in the physical meaning of the parameters. The quantities p and q from Eq. (6) characterize the rheological properties of the soil as a set without dividing them into the elastic and viscous components.

The possibility and merits of modeling by Eq. (6) the laws governing the compression of the derno-podzolic easy-loamy soil of a certain granulometric composition at humidity w = 16–26% [9–11] and black-earth soils of Central Povolzhie [12] and of others have been confirmed experimentally.

In the formulas obtained on the basis of (6), it is expedient to represent the characteristic p as $p = g\omega$, where ω is the frequency of the harmonic process of deformation arising, in particular, due to the loading of the soil in rolling of a cylindrical die or a wheel. The characteristics of the viscoelastic properties of the soil g and q can be found from processing the experimental data by the technique of [10]. For investigation of the derno-podzolic easy-loamy soil we suggest the following linear regression equations obtained by processing the experimental data (at $\rho_d = \rho/(1+0.01w)$ = 1.138–1.579 g/cm^3, w = 16–26%, ω = 0.93–5.01 sec^{-1}):

$$g = 14.655 - 6.716\rho - 0.581\omega + 0.085w \,, \tag{7}$$

$$q = -\,9.654 + 14.981\rho + 0.245\omega - 0.315w \tag{8}$$

with coefficients of multiple correlation of 0.7931 and 0.7528, respectively.

We have studied and modeled mathematically the creep processes of soils whose rheological properties are described by Eq. (6). In this case, we have assumed that the deforming soil layer which spreads to a depth H, lies on a rigid base, e.g., on a soil layer whose deformations are negligibly small. The surfaces of both the deforming soil layer and the rigid base are horizontal. The outer load is transferred to the soil surface via the die.

According to the results of statistical processing of experimental data, the dependence of the density ρ of the deforming soil layer before the action of the outer load on it on the depth y was taken to be linear:

$$\rho\,(y) = \hat{\rho} + ky \,. \tag{9}$$

We considered two stages of variation of the stressed-deformed state of the soil. In the first (initial) stage, at $t \in [0; t_0]$, both deformations and stresses in the soil change, increasing from their zero values to ε_0 and σ_0. In the second stage (creep), at $t \in (t_0, \infty)$, stresses $\sigma = \sigma_0$ = const and relative compression deformation of the soil changes with time $\varepsilon = \varepsilon(t)$. We studied creep processes arising after initial deformation of the soil by the harmonic and linear laws.

Let variation of compression deformations in the first stage occur by the harmonic law

$$\sigma\,(t) = \sigma_m \sin \omega t \,, \quad t \in [0; t_0] \,. \tag{10}$$

Having substituted $d\sigma/dt$ into (6), with account for the initial condition (at $t = 0$ deformation $\varepsilon = 0$) we obtain

$$\varepsilon\,(t) = \frac{\sigma_m}{q}\,(\sin \omega t - g \cos \omega t + g) \,, \quad t \in [0; t_0] \,. \tag{11}$$

The numerical values of the quantities g and q which enter into (11) correspond to $\rho = \hat{\rho}$.

As a result of the action of the outer load, soil deformation, the depth of propagation of the deforming soil layer, and density change with time. Before the beginning of the creep stage, we have $h_0 = h(t_0)$ and $H_0 = H - h_0$.

On the basis of the solution of the boundary-value problems of propagation of viscoelastic damping waves of compression deformations, which arise under the action of an outer load in the soil having variable, linearly dependent on depth, density, we obtained formulas and algorithms which allow one to obtain the depth of propagation of the compression deformation H_{pr} of the soil, the density increment $\Delta\rho(y)$, and the soil density at different depths [11]. In the investigation, the results of which are presented in this paper, we considered the case where the value of H is small, with $H_{pr} = H$. We assume that at the depth H the soil, before the action of the outer load on it, had a maximum possible density; therefore, $\Delta\rho(H) = 0$.

In the second stage of variation of the stressed-deformed state of the soil, we take the coordinate $y = h_0$ as a new origin of depth reckoning $\tilde{y}$ (i.e., the soil surface). At the beginning of the creep process (at $\tilde{y} = 0$) the soil density is

$$\hat{\rho}_0 = \hat{\rho} + kh_0 + \Delta\rho\,(h_0)\,. \tag{12}$$

According to [11], the dependence of the soil density on $\tilde{y}$ can be taken to be linear with a high degree of accuracy. With $\hat{\rho}_0$ and $\rho(H_0)$ being known, we find the dependence of the form (9) of the soil density on $\tilde{y}$. In this case, in Eq. (9) $y = \tilde{y}$, $\rho(y) = \rho_0(\tilde{y})$, $\hat{\rho} = \hat{\rho}_0$, and $k = k_0$.

As the soil density changes, the relaxation properties of the soil, which will be determined by new values of g_0 and q_0 of the characteristics of its viscoelastic properties, which at the beginning of the creep stage correspond to the equality $\rho = \hat{\rho}_0$, will also change. In the creep stage $\sigma = \sigma_0 = \sigma_m$, $\sin \omega t_0$ = const. Substituting $d\sigma/dt = 0$ into (6) and allowing for the initial condition (at $t = t_0$ deformation $\varepsilon = \varepsilon_0$), we obtain the formula characterizing the creep process of the soil:

$$\varepsilon\,(t) = \varepsilon_0 + \frac{g_0\omega}{q_0}\,\sigma_0 t\,, \quad t \in (t_0, \infty)\,. \tag{13}$$

The quantity ε_0 which enters into (13) is determined by (11) at $t = t_0$.

At constant values of the parameters Eq. (13) describes linear creep typical of the ideal viscoelastic Maxwell media, but not of a real soil. When $\sigma_0 < \sigma_{st}$, its deformation stabilizes with time.

The values of the compression deformation of the soil, the depth of the deforming soil layer, and characteristics g and q of its viscoelastic properties, which change during the creep process until the onset of deformation stabilization, are continuous functions of time. It is approximately assumed that in small time intervals they are constant, with their changes occurring stepwise. We developed an algorithm for determining, during small intervals of time $\Delta t_s = t_s - t_{s-1}$ (s = 1, 2, ..., N), the increments of the relative $\Delta\varepsilon_s$ and absolute Δh_s compression deformations of the soil, increments of soil density at different depths, characteristics q_s and g_s of the viscoelastic properties of the soil, total settlement of the soil h_s at $t = t_{s-1}$, depth of the deforming soil layer H_s, and factors of the stressed-deformed state of the soil at different fixed instants of time.

As time goes on, the characteristic q_s increases and g_s decreases; here $\Delta h_s \to 0$, $h_s \to h_{stab}$ = const. The time during which h_s reaches h_{stab} is the time of deformation stabilization t_{stab}. From the calculations we obtain: at $\Delta t_s = \Delta t_N$ the increment of settlement $\Delta h_N \approx 0$, the total settlement of the soil $h_N \approx h_{stab}$ and $t_N \approx t_{stab}$.

We developed computer software which allows one, using the obtained formulas and algorithms, to determine the factors characterizing the stressed-deformed state and the density of the soil at different fixed instants of time during its loading at $t \in [0; t_0]$ and in the creep process. With this software we calculated the factors mentioned for the derno-podzolic easy-loamy soil of the known granulometric composition at a soil humidity of w = 16–24%, having assumed H = 0.51 m.

We studied the effect of the initial density and humidity of the soil, rate of change of stresses in loading, and time t_0 on creep and compaction of the soil. To reveal the character and to obtain the quantitative estimate of the effect of these factors, we conducted a series of calculations (computer experiments).

We determined the increments $\Delta\varepsilon_s$ and Δh_s in time intervals Δt_s, the total values of ε_s and h_s at different fixed instants of time $t_s \in [0; t_N]$ ($t_N \approx t_{stab}$), and also the corresponding values of $\Delta\rho(\Delta h_s)$ and density $\rho_0(0.05)$ of the soil, the parameters of the linear dependence of the density of the soil on the depth and the characteristics of its viscoelastic properties. As approximate values of t_{stab} we took the instants of time to which values $\Delta\varepsilon_s \le 10^{-5}$ corresponded.

We conducted eight series of one-factor experiments and two series of complete three-factor experiments (numerical computer experiments). These experiments were conducted for two regimes of variation of stresses at $t \in [0; t_0]$: by the harmonic law (10) at t_0 = 0.1 sec and ω = 3.15 sec^{-1} and by the linear law $\sigma = vt$ ($v > 0$ = const) at t_0 = 0.1 sec and v = 1.2 m/sec.

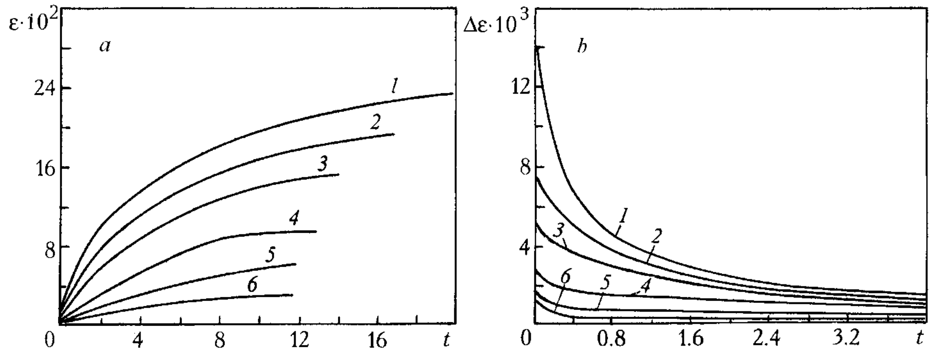

Fig. 2. Dependence of relative compression deformation of the soil (a) and its increment (b) on time from the onset of creep at different values of initial density (w = 19%; σ_m = 36 kPa; ω = 3.15 sec^{-1}; t_0 = 0.1 sec): 1) $\hat{\rho}$ = 1.1 g/cm^3 and k = 1.7647 g/(cm^3·m); 2) 1.2 and 1.5686; 3) 1.3 and 1.3725; 4) 1.5 and 0.9804; 5) 1.7 and 0.5882; 6) 1.9 and 0.1960. t, sec.

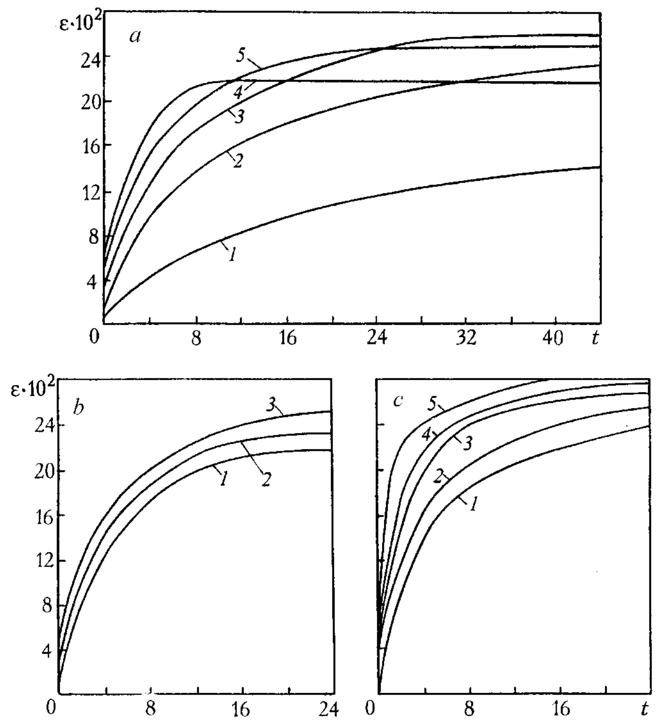

Fig. 3. Creep of the derno-podzolic easy-loamy soil at different values of rate of initial loading of the soil (a) [1) ω = 0.9 sec^{-1}; 2) 2; 3) 3; 4) 4; 5) 5.4 (t_0 = 0.1 sec, w = 19%)], humidity of the soil (b) [1) w = 16%; 2) 20; 3) 22 (t_0 = 0.1 sec, ω = 3.15 sec^{-1})], and time of initial loading (c) [1) t_0 = 0.1 sec; 2) 0.15; 3) 0.25; 4) 0.3; 5) 0.5 (w = 19%, ω = 3.15 sec^{-1})]; ρ = 1.1 g/cm^3; k = 1.7647 g/(cm^3·m); σ_m = 35 kPa. t, sec.

In each series of one-factor experiments, one of the four main affecting factors were varied: (1) initial density of the soil; (2) frequency ω or linear velocity v of variation of stresses at $t \in [0; t_0]$; (3) humidity of the soil w, or (4) time t_0.

We revealed the substantial influence of the quantities $\hat{\rho}$, w, and ω or v and t_0 on the change of $\sigma(t)$ and $\varepsilon(t)$ and of other factors with time.

Figures 2 and 3 give graphs which reflect the results of some experiments. The obtained dependences $\varepsilon(t)$ show that at the same value of σ_0 to higher values of the initial density of the soil there correspond smaller values of

ε_0. As $\hat{\rho}$ increases, the quantities $\varepsilon_{\lim}$, $\Delta\rho(h_0)$, and t_{stab} decrease. With increase in the rate of loading at $t \in [0; t_0]$ the deformation of the soil increases and the quantities $\varepsilon_{\lim}$ and t_{stab} decrease. An increase in $d\sigma/dt$ at $t \in [0; t_0]$ leads to compactness of the soil.

At the same compression stresses, deformations $\varepsilon(t)$ in moister soil are more substantial. As the humidity of the soil increases, the quantities ε_0, $\varepsilon_{\lim}$, $\Delta\rho(h_0)$, and t_{stab} also increase.

Stresses σ_0 increase with time t_0; therefore, the curves $\varepsilon(t)$ shown in Fig. 3c characterize the influence of σ_0 on creep. The results obtained show that an increase in t_0 (and correspondingly σ_0) causes an increase in the quantities $\varepsilon(t)$ (when $t < t_{stab}$), $\rho_0(h_0)$, and $\varepsilon_{\lim}$.

The results of complete factor computer experiments were used for obtaining correlation dependences of compression stresses in the soil and of other factors on $\hat{\rho}$, w, and also ω or v at different fixed instants of time $t \le t_{stab}$. The coefficients of the regression equations, their significance, and the adequacy of the regression equations were determined by the technique of [13]. Verification by the Fischer criterion at the 5% significance level showed that the obtained regression equations are adequate (i.e., are suitable for description of experimental data). An analysis showed that almost all the studied factors are affected by the initial density of the soil and the rate of deformation. The mean values of the quantities ε_0, $\varepsilon(t)$, and $h(t)$ decrease as $\hat{\rho}$ increases, whereas with increase in ω they increase.

The obtained regression equations indicate that the effect of soil humidity within the considered range of its variation is much smaller than the effect of $\hat{\rho}$ and the rate of deformation at $t \in [0; t_0]$. It is revealed that all the factors considered increase with w.

The indicated character of the effect of the initial density and humidity of the soil on its creep and compactness stem from the following fact. As $\hat{\rho}$ increases, the characteristic q increases, with $q \to E$, and the characteristic g decreases, with $g \to 0$. In this case, the properties of the soil approach elastic ones. With increase in the humidity w of the soil, the characteristic g of its viscoelastic properties increases and the characteristic q decreases, with $q \to 0$. In this case, the elasticity of the soil decreases and the properties of the soil approach flowing ones.

The obtained results of the study of the process of creep of soils are in agreement with the experimental data from the works dealing with the creep processes in soils [3, 6–8, 11, 12] and other deformable media [5].

CONCLUSIONS

1. We suggested and substantiated mathematical models of deformation of compacting soils under different conditions of their loading: the integral equation (1) with kernel (2) and the differential equation (6).

2. It is shown that (1) very accurately models the laws governing the deformation of soils of various granulometric compositions with time within a wide range of variation of their density and humidity.

3. At rather small values of t and ε, the integral equation (1) can, in approximation, be replaced by the differential equation (6). The possibilities and merits of modeling of the laws governing the compression of a number of soils by Eq. (6) are confirmed experimentally.

4. The interrelation between two different types of mathematical models of the laws governing deformation of soils — the equations which relate deformations and stresses not involving time and the equations which describe the rheological properties of soils — is revealed.

5. The interrelation of the rheological models of deformation of soils — integral and differential equations — are substantiated.

6. On the basis of mathematical modeling of the viscoelastic properties of soils by the differential equation (6) we obtained analytical relationships and algorithms which allow one to find (by computation) the factors of the stressed-deformed state of the soil under different conditions of loading with account for the time factor and in creep and also to determine the increment of the soil density at different depths which arises under the action of the outer load.

7. As a result of computer experiments, we found the factors characterizing the creep of the derno-podzolic easy-loamy soil and its compactness and strengthening in loading according to different laws and in the creep process.

8. Correlation dependences of the factors studied on the initial density of the soil, its humidity, and the rate of deformation at $t \in [0; t_0]$ are revealed. These dependences allowed estimation of the effect of the most important factors on the occurrence of relaxation processes in the soil.

9. The results obtained can be used in the development of a complex of measures aimed at conserving and improving the fertility of soils.

NOTATION

σ, compression stress, MPa; h, absolute compression deformation, m; ε, relative compression deformation; t, time, sec; w, weighting (absolute) humidity of the soil, %; w_{tot}, total moisture capacity of the soil, %; τ, current time preceding the time instant t, sec; $K(t)$, kernel of the integral equation; A, α, and β, parameters of the kernel of the determining integral equation (1) for the soil; $\Gamma(\alpha)$, gamma-function; n, ordinal number of the term in the functional series in formula (2); $\sigma = \varphi_j(\varepsilon)$, equations of isochrones; $\sigma = \varphi_0(\varepsilon)$, equation of the curve of instantaneous deformation (isochrone corresponding to $t = 0$); λ_j, similarity coefficients; δ, mean value of relative deviations of the calculated data from those obtained experimentally; σ_{st}, strength limit of the soil, MPa; p and q, parameters of the determining differential equation (6) for the soil (characteristics of viscoelastic properties of the soil), MPa and sec^{-1}; ω, frequency of the harmonic process of deformation, sec^{-1}; $g = p/\omega$, transformed dimensionless characteristic of viscoelastic properties of the soil; y and $\tilde{y}$, vertical coordinates of a particle of the deforming soil layer before soil loading (depth) and in the creep process of the soil, m; H, depth of propagation of the deforming layer of the soil before its loading, m; ρ and ρ_d, densities of moist and absolutely dry soils, g/cm^3; $\hat{\rho}$ and $\hat{\rho}_0$, density of the soil before its loading at $y = 0$ (on the soil surface and at the beginning of the creep process at $\tilde{y} = 0$, g/cm^3; k, angular coefficient of the straight line (9), g/(cm^3·m); σ_m, stress amplitude under loading by the harmonic law, MPa; H_{pr}, depth of propagation of the compression, deformation of the soil, m; k_0, angular coefficient of the straight line $\rho_0(\tilde{y})$ at the beginning of the creep process, g/(cm^3·m); Δ, increment of the quantity; v, linear velocity, m/sec; E, elasticity modulus, MPa. Indices: tot, total (moisture capacity); exp, experimental values; mean, mean value; st, strength; stab, stabilization of deformation; d, dry soil; m, maximum value; pr, propagation of deformation; lim, limiting value; i and r, number of the creep curve and their quantity in the set ($i = 1, 2, ..., r$); j, number of the isochrone corresponding to the fixed instant of time t_j; s and N, number of the fixed instant of time and the quantity of such instants of time in consideration of creep and compactness of the soil in creep ($s = 1, 2, ..., N$); 0, values of σ, h, ε, t, H, g, q, and k at the beginning of the creep process.

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